# **Unstable cracking of medium density polyethylene**

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The ductile instability characterization of medium density polyethylene (MDPE) has been studied with the J integral R curve method. The loading of variously notched samples in three-point bending using a compliant test fixture showed that sudden unstable cracking occurred after the commencement of stable ductile cracking. The tearing modulus which is obtained from the  $R$  curve can describe the transition from stable to unstable crack. Instability occurs when an applied situation depending on loading system and/or sample geometry exceeds the tearing modulus of MDPE.

## **1. Introduction**

There has been extensive work over recent years on the short-term and long-term fracture mechanism of medium density polyethylene (MDPE), especially, as this material is widely used for pipes to transport gases. As far as short-term fracture is concerned, MDPE is a tough material which usually fails in a ductile manner, exhibiting considerable necking in a tensile test at room temperature. Therefore, a crack in MDPE grows in a stable manner with the forming of a large plastic deformation zone ahead of a crack tip. However, under loading conditions or specimen geometries and sizes in which more strain energy is released than can be absorbed by deformation processes near the crack tip, the excess energy can accelerate the crack to uncontrollable speeds even in a ductile material such as MDPE. Although this may appear to be similar to brittle crack growth, it is called "ductile tearing instability" or "energetic instability" because the crack growth is essentially ductile but unstable [1].

This instability phenomenon sets limits to the design and safety of the gas pipes. There has been much effort devoted to studying the time-dependent slow crack growth which finally results in brittle fracture [2-4], but little attention has been paid to instability in MDPE. The objective of this investigation is to clarify the loading conditions and specimen geometries which enhance unstable cracking, and to obtain a useful criterion for predicting the instability in MDPE for gas pipe use.

## **2. Material and specimens**

The material used in this experiment is medium density polyethylene commercially available for gas pipe use. The density is  $935 \text{ kg m}^{-3}$ ; MFI = 0.19; the molecular weight is 123 000. The resin was compression moulded into 10 mm thick plaques. Rectangular specimens of 15 mm width and 80 mm length were cut from these plaques. The elastic modulus and yield stress of these specimens were 645 and 12.9 MPa, respectively. A U notch of 4.0 mm length was machined in the samples and then a sharp notch of 1.0 mm depth was introduced by pressing a razor blade for the J integral test. Similarly, sharp notches of various depths, *a/H,* from 0.30 to 0.57, were made for the instability tests.

### **3. The J integral**

The specimens were loaded in three-point bending with a span length of 60mm in a universal testing machine at a cross-head speed of  $5.0 \text{ mm min}^{-1}$ . All tests were carried out at 296 K and 50% relative humidity. A typical bending moment against displacement curve is shown in Fig. 1. The plastic deformation initiated around the crack tip at a stress level of nearly 40 to 50% of the maximum bending moment. As the plastic deformation grew ahead of the crack tip, crack tip blunting occurred to form a crack opening stretch prior to actual crack growth. The method used here to obtain the  $J$  integral is the multiple specimen method called the  $R$  curve method [5], i.e. loading a series of identical specimens to different loading points and then unloading. The values of the  $J$  integral are calculated by

$$
J = 2U/bB \tag{1}
$$

where  $U$  is the area under the load against load-point displacement curve up to the unloaded point,  $B$  is the specimen thickness, and  $b$  is the ligament length at the cracked section. After unloading, the specimens were cut and thin sections were prepared for microscopic measurements of the amount of crack growth,  $\Delta a$ . Fig. 2 shows the relationship between the J values and



*Figure* 1 A typical load aginst deflection curve and crack growth for MDPE with an initial crack length of 0.5.

 $\Delta a$  obtained. This relationship is usually called the material resistance curve  $(R$  curve) because the slope of the curve, *dJ/da,* is related to the energy release rate required for stable crack growth.

There are several techniques for metals to detect the actual crack growth for determining a critical J value,  $J<sub>lc</sub>$ , from the R curve. A simple one is given by the ASTM E813 standard in which  $J_{\text{lc}}$  is graphically found by specifying the intersection of the R curve and blunting line. When a blunted crack is assumed to have a semi-circular profile, the blunting line is given by

$$
J = 2\sigma_0 \Delta a \tag{2}
$$

where  $\sigma_0$  is the yield stress of the material. It is experimentally well established in elastic-plastic fracture of metals that the point of intersection of the R curve and the blunting line given by Equation 2 describes the critical J value for the onset of slow crack growth. However, in polymers, there still remains some doubt about this graphical method because of the complexity of plastic deformation which is composed of a combination of local shear yielding, crazing and voiding. Although there is generally no significant difference between the J integral obtained by extrapolating the  $R$  curve and the blunting line, more detailed measurements at the stage of initial cracking are needed to apply the ASTM method to ductile polymers. This has been discussed by one of the authors elsewhere [6, 7].



*Figure 2 J* integral R curve for MDPE against slow crack extension. , blunting line.



*Figure* 3 A schematic showing a compliant test fixture.

### **4. The instability criterion**

A crack begins to grow at a critical  $J$  value which equals the material resistance at crack initiation. It continues to grow in an equilibrium manner if this condition is maintained, that is

$$
J(P, a) = R(a) \tag{3}
$$

where  $J$  is a function of the applied load,  $P$ , and crack length, and  $R$  is the material resistance to crack growth and only a function of crack length as experimentally shown in Fig. 2. Therefore, as the material resistance  *increases with increasing crack length the* applied load must normally increase for continuous equilibrium cracking.

Paris and coworkers [8, 9] have provided a simple analysis of the mechanics of the instability criterion associated with the  $J$  integral and  $R$  curves. Simply consider a centre-cracked strip with crack length 2a, width  $W$ , thickness  $B$ , and length  $L$ . When fully plastic yielding is assumed at the ligament section, the limit load  $P$  is given by

$$
P_{\rm L} = \sigma_0 (H - 2a) B \tag{4}
$$

With crack extension,  $da = -db$ , the limit load is reduced

$$
dP = 2\sigma_0 daB \tag{5}
$$

and this load reduction causes elastic recovery of the length of the specimen

$$
d(\Delta Le) = dP_L L/AE = (-2\sigma_0 da L)/WE
$$
 (6)

where  $E$  is the Young's modulus of the material. At the same time, the crack growth increases the length of the specimen due to increasing crack opening stretch,  $\delta$ <sub>T</sub>, that is

$$
d(\Delta L p) = d(\delta_{\mathrm{T}}) \tag{7}
$$



*Figure 4* The effect of loading compliance on the load against deflection curves of MDPE. O, instability.



*Figure 5* The effect of initial crack length on the load-deflection curves of MDPE. o, instability.

an increase in  $\delta_{\rm T}$  gives a corresponding increase in the J integral

$$
d\delta = dJ/da \qquad (8)
$$

Combining Equations 7 and 8 gives

$$
d(\Delta L p) = dJ/\sigma_0 \tag{9}
$$

If the specimens were being tested in a rigid machine (fixed grips and end displacement) instability occurs when the magnitude of elastic shortening exceeds the corresponding plastic lengthening required for crack growth. Then, equations 5 and 8 lead to a criterion for instability

$$
\frac{\mathrm{d}J E}{\mathrm{d}a \sigma_0^2} < \frac{2L}{W} \tag{10}
$$

Similarly for a three-point bending specimen, the instability criterion is given by

$$
\frac{\mathrm{d}J}{\mathrm{d}a} \cdot \frac{E}{\sigma_0^2} = \frac{\mathrm{d}R}{\mathrm{d}a} \cdot \frac{E}{\sigma_0^2} < \frac{2b^2S}{H^3} - \theta E \tag{11}
$$

where S is the span length and  $\theta$  the rotational angle of the bending specimen. The left-hand side of Equations 9 and 10 is expressed as  $T_{\text{mat}}$  "tearing modulus", which depends only on material properties. The right-hand sides of Equations 9 and 10 are nondimensional parameters,  $T_{\text{appl}}$ , which depend mainly on the specimen configuration if a rigid machine is used for testing.

### **5. Instability testing**

The criterion for instability given by Equations 10 or 11 assumes a test using a rigid machine and rigid fixtures. Adding compliance to the test arrangement by using a spring bar in the test fixtures is equivalent to increasing length of the test specimen, L or S. The most compliant machine is a load-controlled situation. The equivalent length is given by

$$
S_{\text{equiv}} = S \left( 1 + \frac{\delta_{\text{SB}}}{\delta_{\text{TB}}} \right) \tag{12}
$$

where  $\delta_{SB}$  and  $\delta_{TB}$  are the elastic deflection of the spring bar and test specimen under the same load, respectively. The equivalent length should be used to replace  $S$  in the instability Equation 11. This implies that the value of  $T_{\text{appl}}$  increases with increasing compliance of the spring bar. As shown in Fig. 3, the three-point bending tests were carried out using a spring bar through which the load was applied to the



*Figure 6* The effects of loading situation and tearing modulus on stable  $(O)$  and unstable  $(\bullet)$  cracking of MDPE.

test specimen. The span length of the spring bar can be adjusted to give a variable spring constant. Fig. 4 shows the comparison of a load against displacement curve obtained by using a rigid test fixture with a compliant one where test specimen dimensions including *a/H* are held nearly constant. An unstable crack starts at the beginning of the steep descending load portion of the load against displacement curve, whereas with stable behaviour no sudden load drop occurs. If the spring compliance is held constant,  $T_{\text{appi}}$ increases with decreasing *a/H* as expected from the instability condition, Equation 11. Fig. 5 shows the effect of *a/H* on the load against displacement curves. The descending portion of the curves becomes steeper with decreasing  $a/H$ . Fig. 6 shows the results of stable and unstable crack propagation obtained by changing spring compliance and  $a/H$ . The value of  $T_{\text{mat}}$  was calculated by using the  $J$  integral data against crack growth shown in Fig. 2. Lying on either side of  $45^\circ$  line is stable or unstable behaviour predicted by Equation 11. It is clearly seen from Fig. 6 that the tearing modulus,  $T_{\text{mat}}$  can well characterize the stable to unstable transition behaviour of ductile polyethylene specimens.

#### **6. Further discussion and conclusions**

The instability behaviour of medium density polyethylene has been clarified by the  $J$  integral  $R$  curve method. The tearing modulus,  $T_{\text{mat}}$ , can well describe the transition from stable to unstable crack. It has been clearly shown that instability occurs when  $T_{\text{app}}$ exceeds  $T_{\text{mat}}$  which is a material constant. The value of  $T_{\text{appl}}$  depends on loading system compliance and/or specimen geometry. However, obtaining a critical value of the tearing modulus and verifying the instability behaviour is quite tedious and expensive as it requires many identical specimens to be tested. What is needed is a simple test to determine the instability point, perhaps using a load-controlled arrangement which can compensate for a sudden load decrease due to crack extension.

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